

# Chaos due to parametric excitation: phase space symmetry and photon correlations

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We discuss dissipative chaos showing symmetries in the phase space and nonclassical statistics for a parametrically driven nonlinear Kerr resonator (PDNR). In this system an oscillatory mode is created in the process of degenerate down-conversion of photons under interaction with a train of external Gaussian pulses. For chaotic regime we demonstrate, that the Poincaré section showing a strange attractor, as well as the resonator mode contour plots of the Wigner functions display two-fold symmetry in the phase space. We show that quantum-to-classical correspondence is strongly violated for some chaotic regimes of the PDNR. Considering the second-order correlation function we show that the high-level of photons correlation leading to squeezing in the regular regime strongly decreases if the system transits to the chaotic regime. Thus, observation of the photon-number correlation allows to extract information about the chaotic regime.

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## I. INTRODUCTION

The nonlinear systems demonstrating chaotic behaviour are still the subject of much attention. In this way many studies have been also made to explore quantum effects in chaotic dynamics Refs. [1–3]. Particularly, there has been a great interest in studying of driven quantum chaotic systems in the presence of dissipation and decoherence, i. e. quantum chaos for open nonlinear systems. The beginning of study of an open chaotic system can be dated back to the papers Refs. [4, 5] where the authors have analyzed the kicked rotor and similar systems with discrete time interacting with a heat bath. At this point, we note that quite generally, chaos in classically conservative and dissipative systems with noise, has completely different properties. For example, strange attractors on Poincaré section can appear only in dissipative systems. On the other hand, the manifestations of chaotic motion in open quantum systems, though widely studied, remain somehow not so clearly understood, both from the mathematical as well as from the physical point of view. One reason is that by including dissipation and decoherence, the computational effort grows drastically, since we have to operate with density matrices instead of wave functions.

The most successful approach that probes quantum dissipative chaos seems to be quantum tomographic methods based on the measurement of the Wigner function, which is a quantum quasi-distribution in the phase space. In this way, the connection between quantum and classical treatment of chaos can be realized by means of a comparison between strange attractors in the classi-

cal Poincaré section and the contour plots of the Wigner functions Refs. [6–8]. On the other hand, alternative methods that probe quantum dissipative chaos involve considerations of entropic characteristics, analysis of statistics of excitation number Refs. [9, 10], and the methods based on the fidelity decay Refs. [11–13] and Kullback-Leibler quantum divergence Ref. [14], and the purity of quantum states Ref. [15].

The simplest physical systems showing quantum dissipative chaos are based on the model of kicked rotator. Its experimental realization, and observation of the models dissipation and decoherence effects are carried out on a gas of ultracold atoms in a magneto-optical trap subjected to a pulsed standing wave Refs. [16], [17]. Recently, the nonlinear Kerr resonator has been used for investigation of quantum chaos at a level of few quanta Ref. [18].

The purpose of this paper is to investigate properties of dissipative chaos for the other class of the simplest oscillatory open systems which possess some remarkable properties, particularly, symmetries in the phase space as well as a nonclassical statistics of excitation numbers. These symmetries can be formulated on the system Hamiltonian and the master equation on one side or on dynamical equations of motion in the semiclassical approach on the other side. If the governing equations are invariant under a symmetric operation to some state variables an interesting question is arises whether this symmetry can be also seen in the chaotic regimes on the system attractors or the Wigner functions. The answer of this question is very important for the deeper understanding of quantum dissipative chaos.

The requirement in realization of this study is to have a proper quantum model showing both chaotic dynamics, symmetry properties and nonclassical statistics of excitation numbers. For this goal, in this paper we propose a parametrically driven nonlinear Kerr resonator (PDNR) operated in the pulsed regime. In this system an oscillatory mode is created in the process of degenerate

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down-conversion of photons under an external Gaussian pulses.

The dynamics of periodically driven nonlinear oscillator has been also studied in Ref. [25]. Note, that an analogous system modeled as a kicked nonlinear system has been proposed for investigation of quantum coherence and classical chaos Ref. [19]. It was also shown, that a more promising realization of this system, including the quantum regime, is achieved in the dynamics of cooled and trapped ions, interacting with a periodic sequence of both standing wave pulses and Gaussian laser pulses Refs. [20, 21]. Recently the experimental realization of quantum chaotic behavior for the model of quantum kicked top on a single atom has been done Ref. [22]. Transition to classical chaos for a system with coupled internal (spin) and external (motional) degrees of freedom has been also considered Refs. [23, 24].

The other important implementation of the PDNR is given by Josephson junction embedded in a continuous linear circuit. Recently, it is consistently demonstrated that the degenerate (as well as non-degenerate) parametric interactions combined with strong Kerr-interaction can be realized in the Josephson junction embedded in a continuous linear circuit Ref. [26]. The continuously driven version of the model was experimentally demonstrated for a microwave cavity with a time-dependent boundary condition realizing by the tunable inductance of a superconducting quantum interference device Ref. [27]. Parametric conversion of microwave photons between two modes in this Josephson parametric converter has already been observed Ref. [28].

It should be noted, that the combined parametrically driven anharmonic oscillator under monochromatic driving has been proposed and studied in the papers Refs. [29], [30] for the special cases without consideration of effects of dissipation. A complete quantum treatment of monochromatically driven parametric oscillator combined with Kerr-nonlinearity has been developed in terms of the Fokker-Planck equation in complex  $P$ -representation Refs. [31], [32]. It has been demonstrated that this system displays two-fold symmetry in the phase space. The cavity mode displays two-phase stability: there are two stable states of mode with equal photon numbers but with different phases, and as a result the Wigner function of mode acquires two-fold symmetry in the phase space (see also Ref. [33]). Note, that two-phase stability has been shown early for monochromatically driven optical parametric oscillator above threshold of generation Refs. [34–36]. Below we show that such symmetry is also displayed in the chaotic regime of PDAO under Gaussian pulses.

The other part of the paper is devoted to investigation of quantum-to-classical correspondence as well as the photon-number correlation for oscillatory mode in chaotic regime. We analyse phenomenon of photon correlation for regular and chaotic regimes of the PDNR on the base of the second-order correlation function at coinciding times that reflects the statistics of the oscillatory

mode. In general, the PDNR mode shows comparatively high-level of photon correlation due to two-photon parametric excitation. As we will show below this level of photon correlation decreases if the system is operated in the chaotic regime. Thus, we conclude that by means of observing the correlator it will be possible to extract information on the chaotic regime. Hence we present a simpler manifestation of quantum chaos directly related to photon correlation.

In the present paper the system under consideration may be modeled classically and quantum mechanically with dissipation included in the both cases. The classical dynamics of this system exhibits a rich structure of regular and chaotic motion, with the parameters of the train of Gaussian pulses being the control parameters. The quantum regime of the PDNR requires a comparatively high level of third-order, Kerr nonlinearity with respect to dissipation, i.e.  $\chi/\gamma > 1$ , that is considered in this paper.

The paper is arranged as follows. In Sec. II we describe the PDNR under pulsed excitation focusing on the phase space symmetry properties of the system. In Sec. III, we study the properties of chaos on base of the Poincaré section, the Wigner functions and the second-order correlation functions. We summarize our results in Sec. IV.

## II. PULSED THE PDNR: PHASE-SYMMETRY

We start with the following Hamiltonian to describe one-mode PDAO excited by the train of pulses

$$H = \hbar\omega_0 a^\dagger a + \hbar\chi(a^\dagger a)^2 + \hbar f(t)(\Omega e^{-i\omega t} a^{+2} + \Omega^* e^{i\omega t} a^2) + H_{loss}. \quad (1)$$

Here,  $a^\dagger$ ,  $a$  are the oscillatory creation and annihilation operators,  $\omega_0$  is the oscillatory frequency,  $\omega$  is the mean frequency of the driving field and  $\chi$  is the nonlinearity strength proportional to the third-order susceptibility for the case of Kerr-media. The coupling constant  $\Omega f(t)$  is proportional to the second-order susceptibility and the time-dependent amplitude of the driving field. It consists of the Gaussian pulses with duration  $T$  which are separated by time intervals  $\tau$

$$f(t) = \sum_{n=0}^{\infty} e^{-(t-t_0-n\tau)^2/T^2}. \quad (2)$$

$H_{loss} = a\Gamma^\dagger + a^\dagger\Gamma$  is responsible for the linear losses of oscillatory mode, due to couplings with a heat reservoir operators giving rise to the damping rate  $\gamma$ .

The reduced density operator of oscillatory mode  $\rho$  within the framework of the rotating-wave approximation, in the interaction picture corresponding to the transformation  $\rho \rightarrow e^{-i(\omega/2)a^\dagger at} \rho e^{i(\omega/2)a^\dagger at}$  is governed by the master equation,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_0 + H_{int}, \rho] +$$

$$\sum_{i=1,2} \left( L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right), \quad (3)$$

where  $L_1 = \sqrt{(N+1)\gamma}a$  and  $L_2 = \sqrt{N\gamma}a^\dagger$  are the Lindblad operators,  $\gamma$  is a dissipation rate and  $N$  denotes the mean number of quanta of a heat bath,

$$H_0 = \hbar\Delta a^\dagger a, \\ H_{int} = \hbar\chi(a^\dagger a)^2 + \hbar f(t)(\Omega a^{+2} + \Omega^* a^2), \quad (4)$$

and  $\Delta = \omega_0 - \omega/2$  is the detuning between half frequency of the driving field  $\omega/2$  and the oscillatory frequency  $\omega_0$ . To study pure quantum effects we focus on the cases of very low reservoir temperatures for which the mean number of reservoir photons  $N = 0$ .

We use this system in the regime of strong Kerr nonlinearities with respect to dissipation. Recent progress in circuit QED, superconducting systems and solid-state artificial atoms has been opened up an effective additional path for arranging devices describing by this model. Particularly, the same Hamiltonian (1) (with  $f(t) = 1$ ) describes the case of a Josephson junction embedded in a transmission-line resonator in the case when the effect of the quadratic part of the Josephson potential is taken into account exactly. The  $a^\dagger$  and  $a$  raising and lowering operators in this case describe the normal mode of the resonator+junction circuit. The nonlinearity is found to lead to self-Kerr effects. Then, by replacing the single junction by a SQUID the Kerr coefficient allows the parametric term describing degenerate two-photon excitation in microwave light [26].

### A. Symmetry on the frame of Poincaré section

In semiclassical approach the corresponding equation of motion for the dimensionless amplitude of oscillatory mode has the following form

$$\frac{d\alpha}{dt} = -i[\Delta + \chi + 2|\alpha|^2\chi]\alpha + if(t)\Omega\alpha^* - \gamma\alpha. \quad (5)$$

This equation modifies the standard equation for parametric oscillator with Kerr nonlinearity in the case of pulsed excitation. It is used below for comparison of the results obtained in quantum approach with analogous ones in semiclassical limit.

Note that to calculate the Poincaré section, we have chosen real  $x_0$  and imaginary  $y_0$  parts of the complex amplitude as an arbitrary initial phase space point of the system at the time  $t_0$ . We then define a constant phase map in the  $(X, Y)$  plane by the sequence of the periodically shifted points  $(X_n, Y_n) = (X(t_n), Y(t_n))$  at  $t_n = t_0 + n\tau$  ( $n = 0, 1, 2, \dots$ ). Poincaré section can be obtained based on  $x = \text{Re}(\alpha)$  and  $y = \text{Im}(\alpha)$ . It is evident from Eq. 5 that the system has symmetry properties in

the phase space for  $\alpha \rightarrow (-\alpha)$  replacement. It is obvious that this phase-symmetry is also displayed on the Poincaré section.

### B. Phase-space symmetry on the frame of Wigner functions

It is evident that this symmetry is also realized in quantum picture of the the PDNR in the phase space. Really, considering the transformations:

$$H' = U^{-1} H U, \quad \rho' = U^{-1} \rho U \quad (6)$$

with the unitary operator

$$U = \exp(i\theta a^\dagger a) \quad (7)$$

we verify that the interaction Hamiltonian (4) satisfies the commutation relation

$$[H, U] = 0, \quad (8)$$

if the parameters  $\theta$  are chosen as  $\theta = \pi$ . The analogous symmetry takes place for the density operator of oscillatory mode

$$[\rho(t), U] = 0. \quad (9)$$

One of the most important conclusions of such symmetries relates to the Wigner functions of the oscillatory mode

$$W(\alpha) = \frac{1}{\pi^2} \int d^2\gamma Tr \left( \rho e^{\gamma a^\dagger - \gamma^* a} \right) e^{\gamma^* \alpha - \gamma \alpha^*}. \quad (10)$$

In this formula, we perform rotations by the angle  $\theta$  around the origin in the phase spaces of complex variables  $\alpha$  corresponding to the field operators  $a$  in the positive P-representation. Indeed, in the polar coordinates  $r, \theta$  of the complex the phase space plane  $X = (\alpha + \alpha^*)/2 = r \cos \theta$ ,  $Y = (\alpha - \alpha^*)/2i = r \sin \theta$  we derive that the Wigner function displays two-fold symmetry in its rotation around the origin of the phase space

$$W(r, \theta + \pi) = W(r, \theta). \quad (11)$$

It is well known that this symmetry takes place for the degenerate optical parametric oscillator (OPO) reflecting on the phase-locking phenomenon in above threshold regime of OPO. According to phase-locking the mode of sub-harmonic generated in OPO are produced with well-defined two phases Ref. [34]. This situation takes place also for the combined system under consideration due to the quadratic form of nonlinear term in the Hamiltonian Ref. [32]. We show below that two-fold symmetry is also displayed in the strange attractors for the PDNR in the pulsed regime.

In the following the distribution of oscillatory excitation states  $P(n) = \langle n | \rho | n \rangle$ , the normalized second-order correlation function  $g^{(2)}$  as well as the Wigner functions

$$W(r, \theta) = \sum_{n,m} \rho_{nm}(t) W_{mn}(r, \theta) \quad (12)$$

in terms of the matrix elements  $\rho_{nm} = \langle n|\rho|m\rangle$  of the density operator in the Fock state representation will be calculated. Here the coefficients  $W_{mn}(r, \theta)$  are the Fourier transform of the matrix elements of the Wigner characteristic function.

We solve the master equation Eq. (3) numerically based on the quantum state diffusion method [37]. The applications of this method for studies of the driven nonlinear oscillators and OPOs can be found in Refs. [6, 7, 9, 10, 38, 39].

It should be mentioned that in the limit of very short duration of the pulses, when Gaussian pulses in the formula Eq. 2 can be approximated by  $\delta(t - t_0 - n\tau)$  functions, and for zero detuning the Hamiltonian (4) is coincided with the interaction Hamiltonian of the paper Ref. [19] devoted to study of parametrically driven anharmonic oscillator. In cited paper a double peaked the phase space distribution of oscillatory mode has been proposed. The Eq.(10) of two-fold symmetry in the phase space has verified this conjecture. We demonstrate below that this phase-symmetry of both the classical equation and the master equation for the chaotic dynamics leads to the symmetry of both strange attractors and the corresponding Wigner functions.

### III. PECULIARITIES OF CHAOTIC REGIME FOR THE PDNR

#### A. Poincaré sections and Wigner functions

In this subsection we analyse chaos in the framework of both semiclassical and quantum probability distributions by using a correspondence between contour plots of the Wigner function and the Poincaré section. In semiclassical limit chaos is observed on the Poincaré section that is constructed from the semiclassical dimensionless position  $x = \text{Re}(\alpha)$  and momentum  $y = \text{Im}(\alpha)$  variables as solutions of Eq. 5 at fixing points in the phase space at a sequence of periodic intervals. Choosing  $x_0$  and  $y_0$  as an arbitrary initial the phase space point of the system at the time  $t_0$ , we define a constant phase map in the plane by the sequence of points at  $t_n = t_0 + n\tau$ , where  $n = 0, 1, 2, \dots$ . This means that for any  $t = t_n$  the system is at one of the points of the Poincaré section. If the PDNR is driven by the sequence of short pulses the dynamics of the system is nonstationary hence the Poincaré section depends on the initial time-interval  $t_0$ . We choose various initial time  $t_0$  in order to ensure that they match to the corresponding time-intervals of the Wigner function.

The typical results of calculations in the semiclassical approach are depicted in Fig. 1 for the mean excitation number  $|\alpha|^2$  and the Poincaré section for the parameters  $\Delta/\gamma$ ,  $\chi/\gamma$ ,  $\Omega/\gamma$  as well as the parameters of pulses corresponding to the chaotic regimes. As we see,  $|\alpha|^2$  shows the usual chaotic dynamics (see, Fig. 1(a)) while the Poincaré section (see, Fig. 1(b)) displays structure of the strange attractor. The Poincaré section is symmetric

over rotation on  $\theta$  by the angle around the origin in the phase-spaces of complex variable for  $\alpha \rightarrow (-\alpha)$ .

Demonstration of chaos in the quantum treatment is presented in Fig. (2) by means of the mean excitation numbers (photon numbers), the contour plots of the Wigner function, time-evolution of the quantum purity and the distribution of oscillatory excitation numbers. Fig. 2(a) depicts the ensemble averaged mean oscillatory number. It is easy to see that while the classical result (see, Fig. 1(a)) shows the usual chaotic behavior, its quantum ensemble counterpart (see, Fig. (2(a))) has clear regular behavior. These results indicate the well known result that quantum dissipative chaotic dynamics is not evident in the mean oscillatory number and quantum chaos can be displayed in the Wigner function. In fact, it is easy to observe in Fig. 2(b) that the contour plots of the Wigner function are generally similar to the corresponding classical Poincaré section. Note that the ensemble-averaged mean oscillatory excitation number and the Wigner functions are nonstationary and exhibit a periodic time-dependent behavior, i.e., they repeat the periodicity of the driving pulses at the over-transient regime. We conclude that the Wigner function reflects the phase symmetry of the strange attractor.

Fig. 2(c) shows the distribution function of oscillatory excitation number  $P(n) = \langle n|\rho|n\rangle$ . It was demonstrated Ref. [9] that in transition from regular to chaotic dynamic of a driven anharmonic oscillator  $P(n)$  is broadening. The analogous situation is realized for the PDNR. While  $P(n)$  for regular dynamics is clearly bell shaped and localized in narrow intervals of oscillatory numbers, the distribution for chaotic dynamics is flat topped with oscillatory numbers from  $n = 0$  to  $n_{max} = 40$  (see, Fig. 2(c)). The shape of distributions changes irregularly in dependence from the duration  $T$  and time intervals  $\tau$  between pulses.

Correspondingly in Fig. 2(d) we plot quantum purity versus dimensionless scaled time intervals. This quantity has been used as a measure of the statistical characteristic of states and decoherence. The purity is defined through the density matrix  $\rho$  of the system as  $P = \text{Tr}(\rho^2)$ . This is measure of decoherence, for the pure state  $\text{Tr}(\rho^2) = 1$ . As it is known that chaos affects coherence, so we can recognize chaos within the framework of purity also. Recently, the purity of quantum states has been applied to probe chaotic dissipative dynamics.

Analyzing the obtained results we can easily conclude that system has chaotic dynamics in quantum limit also demonstrating the phase space symmetry. These results are calculated numerically on the base of master equation by means of the quantum state diffusion method and for the same parameters of PDAO as it has been shown in Fig. 1.

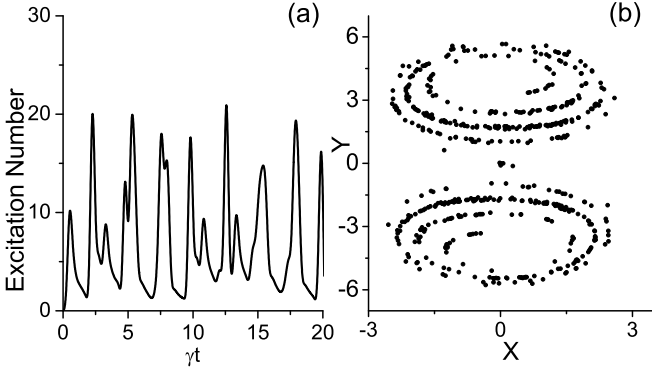


FIG. 1: Semiclassical results for the PDNR: the mean excitation numbers (photon numbers) versus dimensionless scaled time (a); Poincaré section (b). The parameters are as follows:  $\Delta/\gamma = -20$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 20$ , (a)  $T = 0.5\gamma^{-1}$   $\tau = 4\pi/5\gamma^{-1}$ .

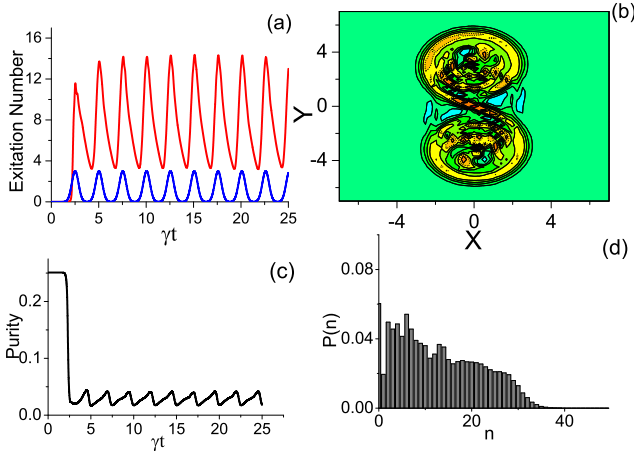


FIG. 2: (Color online) The mean excitation numbers of the PDNR obtained from quantum averaging with snapshots of Gaussian pulses that shows below (a); contour plots of the Wigner function is the phase space (b); time-dependence of the quantum purity (c); distribution of oscillatory excitation numbers (d). The parameters are the same as for Fig. 1:  $\Delta/\gamma = -20$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 20$ , (a)  $T = 0.5\gamma^{-1}$   $\tau = 4\pi/5\gamma^{-1}$ .

### B. Violation of quantum-to-classical correspondence

Another interesting part of our study is devoted to quantum-classical correspondence for the chaotic regime of the PDNR. The connection between quantum and classical treatments of chaos is considered by means of comparison between strange attractors on the classical Poincaré section and the contour plots of the Wigner functions.

As shows results of calculations for the standard pulsed driven nonlinear oscillator Ref. [6], for comparatively small values of the ratio  $\chi/\gamma$ , the contour plots of

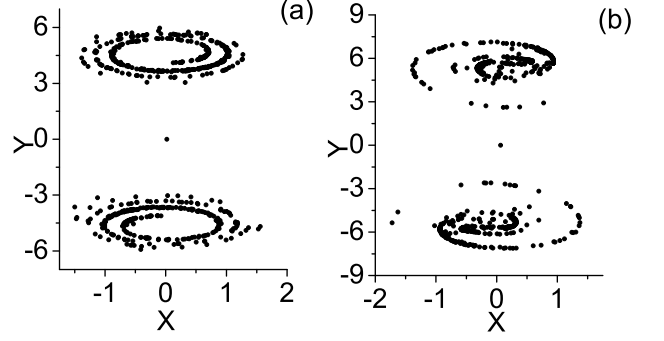


FIG. 3: Poincaré sections for the positive and negative detunings. The parameters are: (a)  $\Delta/\gamma = -10$ ,  $\chi/\gamma = 2$ ,  $\Omega/\gamma = 40$ ,  $T = 0.5\gamma^{-1}$   $\tau = \pi\gamma^{-1}$ ; (b)  $\Delta/\gamma = 15$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 10$ ,  $T = 0.5\gamma^{-1}$   $\tau = 2\pi\gamma^{-1}$ .

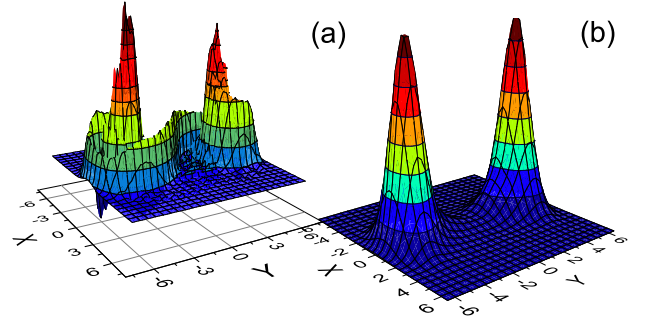


FIG. 4: (Color online) Wigner functions for the positive and negative detunings. The parameters are: (a)  $\Delta/\gamma = -7.5$ ,  $\chi/\gamma = 0.5$ ,  $\Omega/\gamma = 10$ ,  $T = 0.25\gamma^{-1}$   $\tau = 2\pi/5\gamma^{-1}$ ; (b)  $\Delta/\gamma = 15$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 10$ ,  $T = 0.5\gamma^{-1}$   $\tau = 2\pi\gamma^{-1}$ .

Wigner functions are relatively close to the strange attractors. The main difference is that the Poincaré section has fine fractal structure while Wigner function contour plot has not. This is due to the Hiesenberg uncertainty relations which prevent sub-Plank structures in the phase space. Such likeness of quantum and classical distribution vanishes in the deep quantum regime.

As calculations show, the PDNR in the pulsed regime and in the semiclassical approach displays chaotic dynamics for both positive  $\Delta > 0$  and negative  $\Delta < 0$  detunings. The corresponding Poincaré sections in the form of strange attractors are depicted for both cases in Fig. (3). We note, that this result is exactly specific for the pulsed PDNR, as for a standard driven anharmonic oscillator in the pulsed regime a chaotic regime is realized only for the case of negative detunings.

Analyzing the chaotic dynamics of the PDNR in quantum treatment we conclude that quantum chaos is realized only for negative detunings, for the parameters satisfies the following criteria:  $\Delta < 0$ ,  $\Omega \simeq |\Delta|$ ,  $\pi/2 \leq \tau/T \leq 2\pi$ , in contrast to the results obtained in the semiclassical

approximation. The typical results for the Wigner functions corresponding to two cases of detunings depicted in Figs. 4. Comparing Poincaré section (see, Fig. 3(b)) and the Wigner function (see, Fig. 4(b)) calculated for the same parameters with the positive detuning we analyse quantum-classical correspondence. It is easy to conclude that while in the semiclassical approach the PDNR shows chaotic regime in quantum treatment the system displays regular dynamics. Indeed, the Wigner function depicted in Fig. 4(b) has two Gaussian-type peaks and displays a twofold symmetry under the rotation of the phase space by angle  $\pi$  around its origin. For the case of negative detunings the Fig. 3(a) clearly indicates the classical strong attractors with fractal structure that are typical for a chaotic Poincaré section. The Wigner function Fig. 4(a) also reflects the chaotic dynamics, its contour plots in the  $(x; y)$  plane are similar to the Poincaré section in form in contrast to the case of positive detunings. It should be noted that the quantum-classical correspondence is strongly violated for some regimes of the PDNR.

### C. Probe of quantum chaos through photon-number correlations

In this section we discuss the possibility of probing quantum chaos by observing photon statistics on base of the second-order correlation function at coinciding times

$$g^{(2)} = \frac{\langle a^\dagger a^\dagger a a \rangle}{\langle a^\dagger a \rangle^2}. \quad (13)$$

For this goal we calculate correlation function for both regular and chaotic regimes of the PDNR for two groups of the parameters leading to approximately equal excitation numbers for each of the regimes. The correlation function is nonstationary and strongly depends on the properties of the input pulsed driving field. The typical results for  $g^{(2)}$  in dependence of time-intervals are depicted in Figs. 5. Fig. 5(a) corresponds to chaotic regime that is realized for the case of negative detuning, while Fig. 5(b) describes the regular dynamic for positive detuning. As we see these results show drastically different behavior of  $g^{(2)}$  for chaotic and regular regimes of the PDNR. Let us consider this point in more details.

In the regular regime the minimal value of  $g^{(2)}$  approximately equals to 1 and is realized for time intervals corresponding to maximal values of the averaged photon numbers. In the maximums the correlation function describe photon bunching that stipulated by parametric two-photon processes leading to excitation of cavity mode. Indeed, in this case  $g^{(2)} = 5.2$  for the mean photon number  $n = 2.5$ . To illustrate this behaviour we also presentation on Fig. 6(c, d) the results on the Wigner functions for the same parameters. As we see the results on statistics of photons are in accordance with the Wigner functions. Indeed, the Wigner functions showing the regular regime display two localized states of the

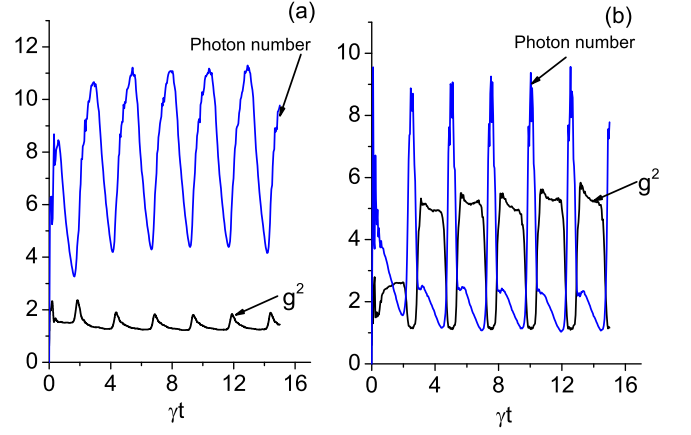


FIG. 5: The averaged photon numbers and the second-order correlation function for both chaotic and regular regimes of the PDNR. The parameters are: (a)  $\Delta/\gamma = -20$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 20$ ,  $T = 0.5\gamma^{-1}$   $\tau = 4\pi\gamma^{-1}/5$ ; (b)  $\Delta/\gamma = 20$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 20$ ,  $T = 0.5\gamma^{-1}$   $\tau = 4\pi/5\gamma^{-1}$ .

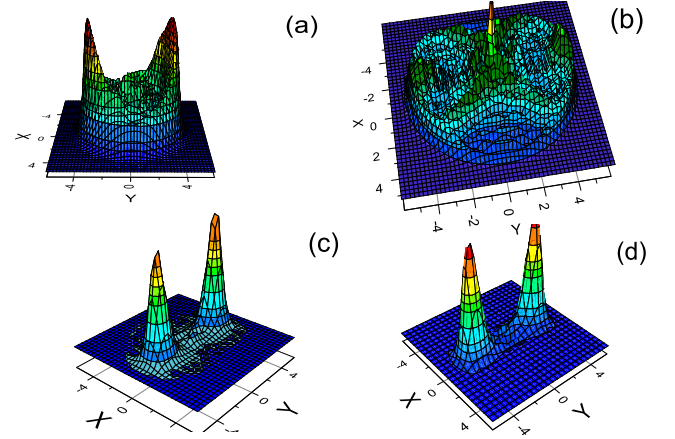


FIG. 6: The Wigner functions for time-intervals corresponding to the minimum and the maximum values of photon numbers depicted in Figs. 5 for the parameters: (a), (b)  $\Delta/\gamma = -20$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 20$ ,  $T = 0.5\gamma^{-1}$   $\tau = 4\pi\gamma^{-1}/5$ ; (c), (d)  $\Delta/\gamma = 20$ ,  $\chi/\gamma = 1$ ,  $\Omega/\gamma = 20$ ,  $T = 0.5\gamma^{-1}$   $\tau = 4\pi/5\gamma^{-1}$ . Figs. (a) and (c) correspond to minimum values of photon numbers, (b) and (d) to the maximum values. The Wigner functions on Figs. (a) and (b) show chaotic behaviour, while Fig. (c) and (d) describe regular regime of the PDNR.

PDNR: for the case of maximal value of photon number they are closer to coherent states, while for the case of photon bunching the peaks are squeezed in the phase space. For the chaotic regime the correlation function at the maximum of photon number approximately equals to 2 that indicates to photon statistics of thermal or chaotic light. For time intervals corresponding to the minimum values of photon numbers  $g^{(2)}$  is slightly decreased. The corresponding Wigner functions depicted in Fig. 6 (a, b)

really confirm the chaotic regime of the PDNR. Thus, we conclude that two-photon processes that stimulate excitation of mode in the PDNR lead to strong photon-number correlation in the regular regime, but have not shown in the chaotic regime. Nevertheless, this peculiarity of the PDNR is displayed as two-fold symmetry in the phase space for both operational regimes.

#### IV. SUMMARY

We have shown pulse designed transition to chaos for the nonlinear dissipative system. It has been demonstrated that nonlinear Kerr resonator parametrically driven by the train of Gaussian pulses shows specific chaotic behaviour, that cardinally different from the analogous results obtained for a standard nonlinear Kerr resonator in the pulsed regime without parametrical excitations. We have shown that the Poincaré sections showing strange attractors in chaotic regimes as well as the contour plots of the Wigner functions of resonator mode display two-fold symmetry in the phase space. This symmetry is naturally formulated as invariable of the system Hamiltonian and the master equation under the symmetry operations. Therefore, in this part of the paper we have confirmed that two-fold symmetry in the phase space is conserved in transition of the PDNR to chaotic

regime.

The classical-to-quantum correspondence in chaotic regime has been considered by means of comparison between strange attractors on the classical Poincaré section and the contour plots of the Wigner functions. In this way, we have demonstrated that in the semiclassical approach chaotic behavior of the PDNR takes place for both positive  $\Delta > 0$  and negative  $\Delta < 0$  detuning in contrast to the case of standard nonlinear Kerr resonator. It has been shown that quantum-to-classical correspondence in the phase space violates for some regimes of the PDNR in the case of positive detunings  $\Delta > 0$ .

We have proposed to probe quantum chaos by observing of photon correlation at coinciding times intervals. In this way, considering the second-order correlation function of photon-number we demonstrate that the high-level of photon correlation leading to squeezing in the regular regime strongly decreases if the system transits to the chaotic regime.

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